Secure communications based on lag synchronization of chaotic complex nonlinear systems with parameters identification

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bstractð Our main goal of this paper is to study secure communications via lag synchronization of chaotic complex nonlinear systems based on adaptive control theory. We try to transmit message from the transmitter system to the receiver system. The transmitted message is modulated into the parameter of the chaotic complex system which consider as the transmitter system. It is assumed that the parameter of the receiver system is unknown. Based on the adaptive control theory the controllers are designed to synchronize two identical chaotic complex Chen systems with unknown parameter as an example. Thus, the uncertain parameter of the receiver system is identified. The information signal of the message can be recovered accurately by the estimated parameter. The corresponding theoretical results and numerical simulations demonstrate the validity and feasibility of a secure communication via chaotic complex nonlinear systems based on adaptive control theory.

Keywords Lag synchronization, Secure communication, Complex systems.

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1 INTRODUCTION

In engineering applications, time delay always exists. For example, in the telephone communication system, the voice one hears on the receiver side at time t is the voice from the transmitter side at time $t-\tau$ ($\tau \geq 0$ and it is the lag time). There also exists time lag as the signal transmitted from the transmitter to the receiver end in chaos-communication. Many experimental investigations and computer simulations of chaos synchronization in unidirectional coupled external cavity semiconductor lasers [1-2] have demonstrated the presence of lag time between the drive and response lasers intensities. The similar experiments for chaotic circuits have also demonstrated the complete synchronization (CS), i.e., the states of two chaotic systems remain identical in the course of temporal evolution, is practically impossible for the presence of the signal transmission time and evolution time of response system itself [3].

Therefore, strictly speaking, it is not reasonable to require the response system to synchronize the drive system at exactly the same time. Lag synchronization (LS) means the state of the response system at time t is asymptotically synchronous with the drive system at time $t-\tau$, namely $\lim_{\tau\to\infty} \|\mathrm{x}(t)-\mathrm{y}(t-\tau)\| = 0$, where $\mathbf{x}(t)$ and $\mathbf{y}(t)$ are the states of the response and drive systems, respectively. Thus, LS is more rigorous than CS in practice and CS is a special case of LS when $\tau = 0$.

Since Fowler et al.[4] introduced the complex Lorenz equations, complex systems have played an important role in many branches of physics [5], especially for chaos-communication, where the complex variables (doubling the number of variables) increase the contents and security of the transmitted information [6]. The main idea of chaos communication is to utilize the chaotic signals as carriers for information transmission, and at the receiver end chaos synchronization is employed to recover the information signal. Hence, the synchronization of complex chaotic systems [7-17] has attracted great attention in the last few decades.

Emad E. Mahmoud et al. had investigated LS of hyperchaotic complex nonlinear systems based on active control [13] and passive control [3], respectively, but they did not applied it to secure communications.

The purpose of secure communication is sending a message from transmitter to receiver through chaotic systems. In other words, the message is injected into chaotic systems, transmitted, and then detected and recovered by the receiver. Many types of secure communication schemes have been presented such as chaotic masking, chaotic switching, or chaotic shift keying and chaotic modulation. In chaotic masking, the message which we need to send it is added to a one of chaotic signal in order to hide it, then the signal is transmitted to the receiver. Under certain conditions the message may be recovered at the receiver. In chaos shift keying, the message is supposed to be binary, and it is mapped into the transmitter and the receiver. In chaotic modulation, the message is injected into the states or the parameters of the chaotic system, or is modulated by using an invertible transformation thus the information signal can be recovered by a receiver if the transmitter and the receiver are synchronized [18-23].

Now there are a number of papers [18-22] about secure communications based on real chaotic signal, while the secure communications based on complex chaotic systems has rarely been studied [23]. Especially, there is almost no paper about the secure communications considering time-delay or time lag based on LS. In fact, as the time lag of transmission, it is more suitable to adopt LS to describe the synchronization between the transmitter and the receiver.

Complex Lorenz system is one of the most common complex chaotic systems, and has been used to describe a detuned laser, rotating fluids [5,24-25], disk dynamos [26], etc. Recently, notably the so-called complex Chen and Lü systems are thought to belong to the same class as the Lorenz equation [6,27-30] and have similar properties. Inspired by the above discussions, We consider the chaotic complex Chen system as an example of chaotic complex nonlinear systems to achieve this investigation. The message is modulated directly into the parameter of the chaotic complex Chen. The parameter of the receiver system is assumed to be unknown. The controllers and the parameter update rule are designed and theoretically analyzed based on adaptive control method (different from [3,13] and it is simpler than them).

2 A CHAOTIC COMPLEX NONLINEAR SYSTEM

A complex dynamical system is called chaotic if it is deterministic, has long-term a periodic behavior, and exhibits sensitive dependence on the initial conditions. A chaotic complex attractor is defined as a complex chaotic attractor with one positive Lyapunov exponents. The sum of Lyapunov exponents must be negative to ensure that system is dissipative. It is even more complicated than chaotic real systems and has more unstable manifolds. Due to chaotic complex systems with characteristics of high capacity, high security and high efficiency, it has a broadly applied potential in nonlinear circuits, secure communications, lasers, neural networks, biological systems and so on. Therefore, research on chaotic complex nonlinear systems is extremely important nowadays [14].
Consider the chaotic complex nonlinear system as follow:

$$\dot{\mathbf{x}} = \Phi(\mathbf{x})\mathbf{A} + \mathbf{f}(\mathbf{x}), \qquad (1)$$

where $\mathbf{x} = (x_1, x_2, ..., x_n)^T$ is a state complex vector, $\mathbf{x} = \mathbf{x}^r + j\mathbf{x}^\mathbf{i}$, $\mathbf{x}^r = (u_1, u_3, ..., u_{2n-1})^T$, $\mathbf{x}^i = (u_2, u_4, ..., u_{2n})^T$, $j = \sqrt{-1}$, T denotes transpose, $\Phi(\mathbf{x})$ is $n \times n$ complex matrix and the elements of this matrix are state complex variables, $\mathbf{A}$ is $n \times 1$ vector of system parameters, $\mathbf{f} = (f_1, f_2, ..., f_n)^T$ is a vector of linear or nonlinear complex functions and superscripts r and i stand for the real and imaginary parts of the state complex vector $\mathbf{x}$.
In this paper we study the definition of LS of two identical systems of the form (1) with known parameters by designing a control scheme.

Remark 1. Most of chaotic complex system can be described by (1), such as complex Lorenz, Chen and Lü systems. In order to show the results of our scheme of two identical systems of the form (1) we choose, as an example, the chaotic complex Chen systems which have been introduced and studied recently in our work [6].

The chaotic complex Chen system is:

$$\begin{aligned} \dot{x} &= \alpha(y - x), \\ \dot{y} &= (\gamma - \alpha)x - xz + \gamma y, \\ \dot{z} &= \frac{1}{2}(\overline{x}y + x\overline{y}) - \beta z, \end{aligned} \qquad (2)$$

where $\mathbf{x} = (x_1, x_2, x_3)^T = (x, y, z)^T$, α, β, γ are positive parameters, $x = u_1 + ju_2, y = u_3 + ju_4$ are complex functions, and u_l $(l = 1, ..., 4), z = u_5$ is real function. Dots represent derivatives with respect to time and an overbar denotes complex conjugate variables. The chaotic complex Chen system are a 5-dimensional continuous real autonomous system. System (2) has trivial and non-trivial fixed points. System (2) exhibits chaotic behavior when $\alpha = 42, \gamma = 26$ and $4 < \beta < 6$, for more detail see [6] . In [6] we calculated numerically, by using the Lyapunov exponents, the parameters values at which these chaotic attractors exist see Fig. 1.

In this system the main variables participating in the dynamics are complex. Clearly, if the variables of the system are complex the equations involve twice as many variables and control parameters, thus making it that much harder for a hostile agent to intercept and decipher the coded message. System (2) is used to describe and simulate the physics of detuned lasers and thermal convection of liquid flows.

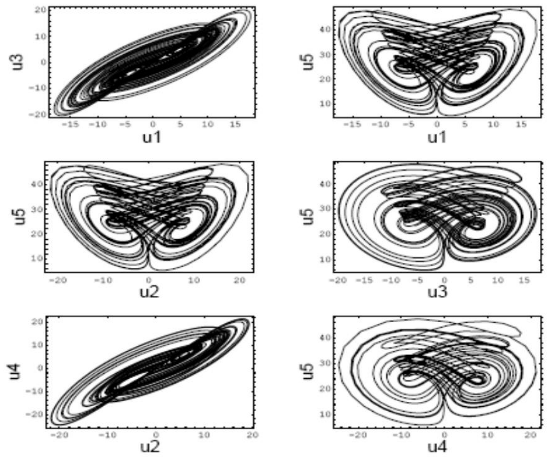

Fig. 1: Chaotic attractors of chaotic complex Chen system in some plans.

3 A SCHEME FOR DESIGN A COMPLEX CONTROLLER OF ADAPTIVE LS

We consider two non-identical chaotic complex nonlinear systems of the form (1), one is the master system (we denote the master system with the subscript m) as:

$$\dot{\mathbf{x}}_m = \dot{\mathbf{x}}_m^r + j\dot{\mathbf{x}}_m^i = \Phi(\mathbf{x}_m)\mathbf{A} + \mathbf{f}(\mathbf{x}_m), \qquad (3)$$

and the second is the controlled slave system (with subscript s) as:

$$\dot{\mathbf{y}}_s = \dot{\mathbf{y}}_s^r + j\dot{\mathbf{y}}_s^i = \Psi(\mathbf{y}_s)\mathbf{B} + \mathbf{g}(\mathbf{y}_s) + \mathbf{L}, \qquad (4)$$

where the additive complex controller $\mathbf{L} = (L_1, L_2, ..., L_n)^T = \mathbf{L}^r + j\mathbf{L}^i$, $\mathbf{L}^r = (v_1, v_3, ..., v_{2n-1})^T$, $\mathbf{L}^i = (v_2, v_4, ..., v_{2n})^T$.

Definition. *Two complex dynamical systems coupled in a master-slave configuration can exhibit LS if there exists a vector of the complex error function* δ *define such as:*

$$\delta = \delta^r + j\delta^i = \lim_{t\to\infty}\|\mathbf{y}_s(t) - \mathbf{x}_m(t-\tau)\| = \mathbf{0}, \quad (5)$$

where $\delta = (\delta_1, \delta_2, ..., \delta_n)^T$, $\mathbf{x}_m(t)$ *and* $\mathbf{y}_s(t)$ *are the state complex vectors of the master and slave systems, respectively,* $\delta^r = \lim_{t\to\infty}\|\mathbf{y}_s^r(t) - \mathbf{x}_m^r(t-\tau)\| = 0$ *and* $\delta^i = \lim_{t\to\infty}\|\mathbf{y}_s^i(t) - \mathbf{x}_m^i(t-\tau)\| = 0$, $\delta^r = (\delta_{u_1}, \delta_{u_3}, ..., \delta_{u_{2n-1}})^T$, $\delta^i = (\delta_{u_2}, \delta_{u_4}, ..., \delta_{u_{2n}})^T$ *, and* τ *is the positive time lag.*

Remark 2. When $\tau = 0$ in Eq.(5) we define compelet synchronization between systems (3) and (4).

Remark 3. If we define $\delta = \lim_{t\to\infty}\|\mathbf{y}_s(t) + \mathbf{x}_m(t-\tau)\|$ and $\tau = 0$ we get CS of systems (3) and (4), while if $\tau > 0$ we obtain anti lag synchronization of the same systems.

Theorem 1. *If nonlinear controller is designed as:*

$$\begin{aligned}\mathbf{L} = \mathbf{L}^r + j\mathbf{L}^i &= -\Psi(\mathbf{y}_s(t))\hat{\mathbf{B}} - \boldsymbol{g}(\mathbf{y}_s(t)) - \Phi(\mathbf{x}_m(t-\tau))\hat{\mathbf{A}} - \boldsymbol{f}(\mathbf{x}_m(t-\tau)) - \mathbf{K}\delta \\ &= -\Psi^r(\mathbf{y}_s(t))\hat{\mathbf{B}} - \boldsymbol{g}^r(\mathbf{y}_s(t)) - \Phi^r(\mathbf{x}_m(t-\tau))\hat{\mathbf{A}} - \boldsymbol{f}^r(\mathbf{x}_m(t-\tau)) - \mathbf{K}\delta^r \quad (6) \\ &+ j[-\Psi^i(\mathbf{y}_s(t))\hat{\mathbf{B}} - \boldsymbol{g}^i(\mathbf{y}_s(t)) - \Phi^i(\mathbf{x}_m(t-\tau))\hat{\mathbf{A}} - \boldsymbol{f}^i(\mathbf{x}_m(t-\tau)) - \mathbf{K}\delta^i],\end{aligned}$$

and the adaptive laws of parameters are selected as:

$$\begin{cases}\dot{\hat{\mathbf{B}}} = (\Psi^r(\mathbf{y}_s(t)))^T\delta^r + (\Psi^i(\mathbf{y}_s(t)))^T\delta^i - \Lambda\tilde{\mathbf{B}}, \\ \dot{\hat{\mathbf{A}}} = (\Phi^r(\mathbf{x}_m(t-\tau)))^T\delta^r + (\Phi^i(\mathbf{x}_m(t-\tau)))^T\delta^i - \Lambda\tilde{\mathbf{A}},\end{cases} \quad (7)$$

then the slave system (4) lag synchronize the master system (3) asymptotically, where $\mathbf{K} = diag(k_1, k_2, \ldots, k_n)$, $\Lambda = diag(\zeta_1, \zeta_2, \ldots, \zeta_n)$, k_l, ζ_l *are positive constants* , $l = 1, 2, ..., n$ *. The parameters of vectors* $\hat{\mathbf{A}}$ *and* $\hat{\mathbf{B}}$ *are the parameters estimation of vectors* $\mathbf{A}$ *and* $\mathbf{B}$ *respectively,* $\tilde{\mathbf{A}} = \hat{\mathbf{A}} - \mathbf{A}$ *and* $\tilde{\mathbf{B}} = \hat{\mathbf{B}} - \mathbf{B}$ *.*

Proof: From the definition of ALS:

$$\delta = \delta^r + j\delta^i = \mathbf{y}_s(t) + \mathbf{x}_m(t-\tau). \quad (8)$$

So,

$$\begin{aligned}\dot{\delta} = \dot{\delta}^r + j\dot{\delta}^i &= \dot{\mathbf{y}}_s(t) + \dot{\mathbf{x}}_m(t-\tau) \\ &= \dot{\mathbf{y}}_s^r(t) + \dot{\mathbf{x}}_m^r(t-\tau) + j[\dot{\mathbf{y}}_s^i(t) + \dot{\mathbf{x}}_m^i(t-\tau)]. \quad (9)\end{aligned}$$

From chaotic complex systems (3) and (4), we get the error complex dynamical system as follows:

$$\begin{aligned}\dot{\delta} = \dot{\delta}^r + j\dot{\delta}^i &= \Psi^r(\mathbf{y}_s(t))\mathbf{B} + \boldsymbol{g}^r(\mathbf{y}_s(t)) + \Phi^r(\mathbf{x}_m(t-\tau))\mathbf{A} + \boldsymbol{f}^r(\mathbf{x}_m(t-\tau)) + \mathbf{L}^r \\ &+ j[\Psi^i(\mathbf{y}_s(t))\mathbf{B} + \boldsymbol{g}^i(\mathbf{y}_s(t)) + \Phi^i(\mathbf{x}_m(t-\tau))\mathbf{A} + \boldsymbol{f}^i(\mathbf{x}_m(t-\tau)) + \mathbf{L}^i]. \quad (10)\end{aligned}$$

Thus, substituting from equation (6) about $\mathbf{L}^r$, $\mathbf{L}^i$ in (10) we obtain:

$$\begin{aligned}\dot{\delta} = \dot{\delta}^r + j\dot{\delta}^i &= \Psi^r(\mathbf{y}_s(t))(\mathbf{B} - \hat{\mathbf{B}}) + \Phi^r(\mathbf{x}_m(t-\tau))(\mathbf{A} - \hat{\mathbf{A}}) - \mathbf{K}\delta^r \\ &+ j[\Psi^i(\mathbf{y}_s(t))(\mathbf{B} - \hat{\mathbf{B}}) + \Phi^i(\mathbf{x}_m(t-\tau))(\mathbf{A} - \hat{\mathbf{A}}) - \mathbf{K}\delta^i], \quad (11)\end{aligned}$$

where vectors of the parameters errors are defined as $\tilde{\mathbf{A}} = \hat{\mathbf{A}} - \mathbf{A}$, $\tilde{\mathbf{B}} = \hat{\mathbf{B}} - \mathbf{B}$. By separating the real and the imaginary parts in Eq. (11), the error complex system is written as:

$$\begin{cases}\dot{\delta}^r = \Psi^r(\mathbf{y}_s(t))(\mathbf{B} - \hat{\mathbf{B}}) + \Phi^r(\mathbf{x}_m(t-\tau))(\mathbf{A} - \hat{\mathbf{A}}) - \mathbf{K}\delta^r, \\ \dot{\delta}^i = \Psi^i(\mathbf{y}_s(t))(\mathbf{B} - \hat{\mathbf{B}}) + \Phi^i(\mathbf{x}_m(t-\tau))(\mathbf{A} - \hat{\mathbf{A}}) - \mathbf{K}\delta^i.\end{cases} \quad (12)$$

For positive parameters, we may now define a Lyapunov function for this system by the following positive definite quantity:

$$\begin{aligned}V(t) &= \tfrac{1}{2}[(\delta^r)^T\delta^r + (\delta^i)^T\delta^i + (\hat{\mathbf{A}} - \mathbf{A})^T(\hat{\mathbf{A}} - \mathbf{A}) + (\hat{\mathbf{B}} - \mathbf{B})^T(\hat{\mathbf{B}} - \mathbf{B})] \\ &= \tfrac{1}{2}\left(\sum_{l=1}^{n}\delta_{u_{2l-1}}^2 + \sum_{l=1}^{n}\delta_{u_{2l}}^2 + \tilde{\mathbf{A}}^T\tilde{\mathbf{A}} + \tilde{\mathbf{B}}^T\tilde{\mathbf{B}}\right). \quad (13)\end{aligned}$$

Note now that the total time derivative of $V(t)$ along the trajectory of the error system (12) is as follows:

$$\begin{aligned}\dot{V}(t) &= (\dot{\delta}^r)^T\delta^r + (\dot{\delta}^i)^T\delta^i + \tilde{\mathbf{A}}^T\dot{\tilde{\mathbf{A}}} + \tilde{\mathbf{B}}^T\dot{\tilde{\mathbf{B}}} \\ &= (\Psi^r(\mathbf{y}_s(t))(\mathbf{B} - \hat{\mathbf{B}}) + \Phi^r(\mathbf{x}_m(t-\tau))(\mathbf{A} - \hat{\mathbf{A}}) - \mathbf{K}\delta^r)^T\delta^r \quad (14) \\ &+ (\Psi^i(\mathbf{y}_s(t))(\mathbf{B} - \hat{\mathbf{B}}) + \Phi^i(\mathbf{x}_m(t-\tau))(\mathbf{A} - \hat{\mathbf{A}}) - \mathbf{K}\delta^i)^T\delta^i \\ &+ \tilde{\mathbf{A}}^T\dot{\hat{\mathbf{A}}} + \tilde{\mathbf{B}}^T\dot{\hat{\mathbf{B}}},\end{aligned}$$

where $\dot{\hat{\mathbf{A}}} = \dot{\tilde{\mathbf{A}}}$ and $\dot{\hat{\mathbf{B}}} = \dot{\tilde{\mathbf{B}}}$. By substituting from Eq. (7) about $\dot{\hat{\mathbf{A}}}, \dot{\hat{\mathbf{B}}}$ in Eq. (14) we obtain:

$$\begin{aligned}\dot{V}(t) &= [\Psi^r(\mathbf{y}_s(t))(-\tilde{\mathbf{B}}) + \Phi^r(\mathbf{x}_m(t-\tau))(-\tilde{\mathbf{A}}) - \mathbf{K}\delta^r]^T\delta^r \\ &+ [\Psi^i(\mathbf{y}_s(t))(-\tilde{\mathbf{B}}) + \Phi^i(\mathbf{x}_m(t-\tau))(-\tilde{\mathbf{A}}) - \mathbf{K}\delta^i]^T\delta^i \\ &+ \tilde{\mathbf{B}}^T[(\Psi^r(\mathbf{y}_s(t)))^T\delta^r + (\Psi^i(\mathbf{y}_s(t)))^T\delta^i - \Lambda\tilde{\mathbf{B}}] \\ &+ \tilde{\mathbf{A}}^T[(\Phi^r(\mathbf{x}_m(t-\tau)))^T\delta^r + (\Phi^i(\mathbf{x}_m(t-\tau)))^T\delta^i - \Lambda\tilde{\mathbf{A}}], \quad (15) \\ &= -[(\mathbf{K}\delta^r)^T\delta^r + (\mathbf{K}\delta^i)^T\delta^i] - \tilde{\mathbf{B}}^T(\Lambda\tilde{\mathbf{B}}) - \tilde{\mathbf{A}}^T(\Lambda\tilde{\mathbf{A}}), \\ &= -\left(\sum_{l=1}^{n}k_l\delta_{u_{2l-1}}^2 + \sum_{l=1}^{n}k_l\delta_{u_{2l}}^2\right) - \tilde{\mathbf{B}}^T(\Lambda\tilde{\mathbf{B}}) - \tilde{\mathbf{A}}^T(\Lambda\tilde{\mathbf{A}}).\end{aligned}$$

Since $V(t)$ is a positive definite function and its derivative is negative definite, thus according to the well-known Lyapunov theorem, the complex error system (10) is asymptotically stable, which means that $\delta_{u_{2l}}$ and $\delta_{u_{2l-1}}$ tend to zero as $t \to \infty$, $l = 1, 2, ..., n$. Consequently, the states of the slave system and the master system will be globally anti-synchronized asymptotically with lag in time. This completes the proof.

Remark 4. If systems (3) and (4) satisfy $\Phi(.) = \Psi(.)$ and $\mathbf{f}(.) = \mathbf{g}(.)$, then the structure of system (3) and system (4) is identical. Therefore, our scheme is also applicable to achieve LS of two identical chaotic complex systems with uncertain parameters.

Remark 5. When systems (3) and (4) are identical $\mathbf{A} = \mathbf{B}$, and the adaptive laws of parameters are selected as:

$$\dot{\hat{\mathbf{A}}} = \dot{\hat{\mathbf{B}}} = [(\Psi^r(\mathbf{y}_s(t)))^T + (\Phi^r(\mathbf{x}_m(t-\tau)))^T]\delta^r$$
$$+[(\Psi^i(\mathbf{y}_s(t)))^T + (\Phi^i(\mathbf{x}_m(t-\tau)))^T]\delta^i - \Lambda\tilde{\mathbf{A}}. \quad (16)$$

Remark 6. When $\dot{\mathbf{x}}_m^i = \dot{\mathbf{y}}_s^i = \delta^i = \mathbf{0}$, our scheme is suitable to achieve LS of two identical or non-identical chaotic systems with ***real*** variables Finally, our scheme is illustrated by applying it for two identical chaotic complex Chen systems in the rest of the work and make application to secure communications.

4 NUMERICAL EXAMPLE

Let us now investigate the LS of two identical hyperchaotic complex Chen systems with uncertain parameters as an example for Section 3. The master and the slave systems are thus defined, respectively, as follows:

$$\begin{aligned} \dot{x}_m &= \rho(y_m - x_m), \\ \dot{y}_m &= (\nu-\rho)x_m + \nu y_m - x_m z_m, \\ \dot{z}_m &= 1/2(\bar{x}_m y_m + x_m \bar{y}_m) - \mu z_m, \end{aligned} \quad (17)$$

and

$$\begin{aligned} \dot{x}_s &= \rho(y_s - x_s) + L_1, \\ \dot{y}_s &= (\nu-\rho)x_s + \nu y_s - x_s z_s + L_2, \\ \dot{z}_s &= 1/2(\bar{x}_s y_s + x_s \bar{y}_s) - \mu z_s + L_3, \end{aligned} \quad (18)$$

where $L_1 = v_1 + jv_2$, $L_2 = v_3 + jv_4$ and $L_3 = v_5$ are complex and real control functions, respectively, which are to be determined.

According to Theorem 1, the controller is designed as:

$$\mathbf{L} = -\Psi(\mathbf{y}_s(t))\hat{\mathbf{B}} - \boldsymbol{g}(\mathbf{y}_s(t)) - \Phi(\mathbf{x}_m(t-\tau))\hat{\mathbf{A}} - \boldsymbol{f}(\mathbf{x}_m(t-\tau)) - k\delta,$$

$$\begin{pmatrix} L_1 \\ L_2 \\ L_3 \end{pmatrix} = \begin{pmatrix} -\rho(y_s(t)-x_s(t)) - \rho(y_m(t-\tau)-x_m(t-\tau)) - k\delta_1 \\ -\hat{\nu} y_s(t) + \alpha_1 - \hat{\nu} y_m(t-\tau) + \alpha_2 - k\delta_2 \\ -\alpha_3 + \hat{\mu} z_s(t) - \alpha_4 + \hat{\mu} z_m(t-\tau) - k\delta_3 \end{pmatrix}, \quad (19)$$

where $\alpha_1 = x_s(t)z_s(t)$, $\alpha_2 = x_m(t-\tau)z_m(t-\tau)$, $\alpha_3 = \frac{1}{2}(\bar{x}_s(t)y_s(t) + x_s(t)\bar{y}_s(t))$, $\alpha_4 = \frac{1}{2}(\bar{x}_m(t-\tau)y_m(t-\tau) + x_m(t-\tau)\bar{y}_m(t-\tau))$, and $\delta_{u_l} = u_{lm}(t-\tau) - u_{ls}(t)$, $l = 1,2,3,4,5,7.$

We can calculate the adaptive laws of parameters by using (16) as:

$$\dot{\hat{\mathbf{A}}} = \dot{\hat{\mathbf{B}}} = \begin{pmatrix} \dot{\hat{\rho}} \\ \dot{\hat{\nu}} \\ \dot{\hat{\mu}} \end{pmatrix} = \begin{pmatrix} (\delta_{u_3} - \delta_{u_1})\delta_{u_1} + (\delta_{u_4} - \delta_{u_2})\delta_{u_2} - \zeta\tilde{\rho} \\ \delta_{u_3}^2 + \delta_{u4}^2 - \zeta\tilde{\nu} \\ -\delta_{u_5}^2 - \zeta\tilde{\mu} \end{pmatrix}. \quad (20)$$

To verify the feasibility of the proposed scheme, we discuss the simulation results of the LS between two identical hyperchaotic complex Lü systems (17) and (18). Systems (17) and (18) with the controller (19) are solved numerically, and the parameters are chosen as $\rho = 40$, $\mu = 4$, $\nu = 22$. The initial condition of the master system state vector, the initial value of the slave system state vector, the positive time lag τ and the diagonal constant matrices are taken as $(x_m(0), y_m(0), z_m(0),)^T = (1+2j, 3+4j, 5)^T$, $(x_s(0), y_s(0), z_s(0),)^T = (6+8j, 3+4j, 8)^T, \tau = 0.2$ and $\mathbf{K} = diag(12,15,11)$, $\Lambda = diag(6,9,10)$. The initial values of estimate for unknown parameters vector are considered as $(\hat{\rho}(0), \hat{\nu}(0), \hat{\mu}(0))^T = (2,3,4)^T$. The results are depicted in Figures 2, 3, 4.
In Figure 2 the solutions of 17 and 18 are plotted subject to different initial conditions and show that LS is indeed with time lag $\tau = 0.1$. Figure 3 shows the numerical simulation of the error δ_{u_l} . Figure 4 shows that the estimated values of the unknown parameters $\hat{\rho}(t)$, $\hat{\nu}(t)$, $\hat{\mu}(t)$ converge to $40, 4, 22$ respectively.

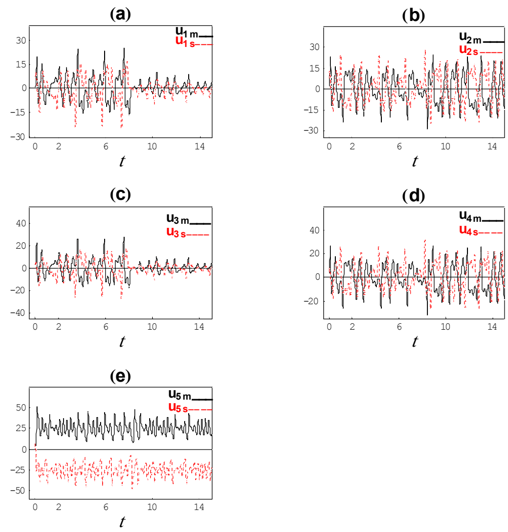

Fig.2: LS between systems (17), (18) via complex controller.

Now we consider system (17) as transmitter system and system (18) as receiver system. For one thing, we choose arbitrarily the information signal as $r(t) = 1 + \sin t$. Take $\breve{r}(t) = r(t) + u_{1m}$ and suppose that $\breve{r}(t)$ is added to the variable u_{2m}. Numerical results of application to secure communication are shown in Figs. 5. The information signal $r(t)$ and the transmitted signal $\breve{r}(t)$ are shown in Fig. 5(a) and (b), respectively. The recovered information signal, which is denoted by $r^*(t) = \breve{r}(t) - u_{1s} - u_{2s}$, is shown in Fig. 5(c). Fig. 5(d) displays the error signal between the original information signal and the recovered one. From Fig. 5(d), it is easy to find that the information signal $r(t)$ is recovered exactly after a very short transient.

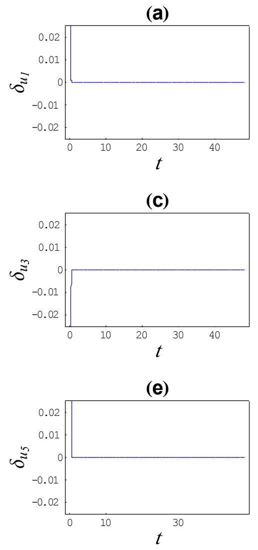

Fig.3: LS errors of systems (17), (18).

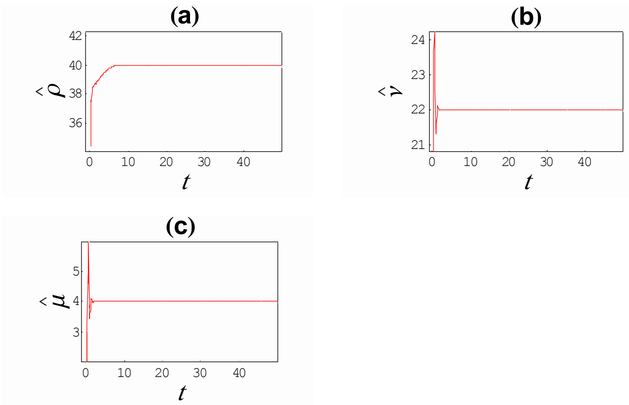

Fig.4: Adaptive parameters estimation laws versus ⍰

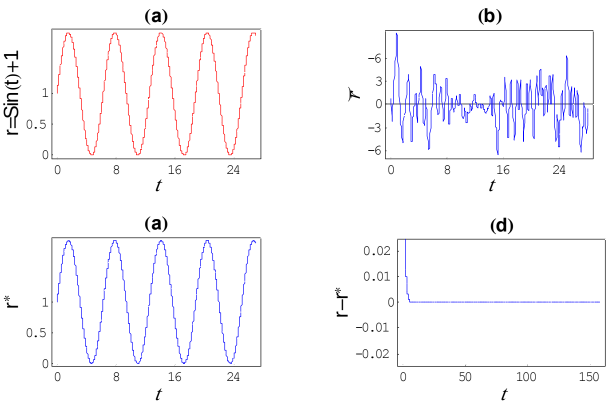

Fig.5: Simulation results of secure communication using LS of two identical chaotic complex Chen systems.

5 CONCLUSION

In this paper we study LS of chaotic attractors of complex systems with uncertain parameters. A scheme is designed to achieve LS of two identical chaotic complex nonlinear systems with uncertain parameters based on Lyapunov functions. Through this scheme we determined analytically the control complex functions and adaptive laws of parameters to achieve LS. The secure communications by using LS in two chaotic complex Chen systems are implemented.

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